Interpolation with Splines Functions

Functions splines are a group of algebraic polynomials greater than or equal to the third degree , which are " linked " together at the interpolation points with values equal to the values of the function f (x) which is interpolated.
Usually in practice we use third degree polynomials , because it is simple to apply and the fact that a low error interpolation. Singular third degree polynomial interpolation functions are called cubic splines.
Let now $x_1, x_2, x_3, \ldots, x_n$ the interpolation points and και $f_1, f_2, f_3, \ldots, f_n$ are the respective values of the function f (x). Let third degree polynomial $P_j(x)$, $x \varepsilon [x_j, x_{j+1}]$ j=1,2,…,n-1 such that

$$P_1(x_1) = f(x_1)$$

$$P_j(x_{j+1}) = P_{j+1}(x_{j+1}) = f(x_{j+1}) \quad j=1,2,\ldots,n-2$$

$$P_n(x_n)=f(x_n)$$

and $P''_1(x_1)=d_1$, $P''_n(x_n)=d_n$ with d_1, d_n known

Suppose

$P''_j(x_{j+1}) = d_{j+1}$, j=1,2,…,n-2 then we

$$P_j^{''}(x) = d_j \frac{x_{j+1} - x}{\Delta x_j} + d_{j+1} \frac{x - x_j}{\Delta x_j} \quad X \in [x_j, x_{j+1}]$$

If we integrate two times P "j (x) we find the relation

$$P_j(x) = \frac{d_j}{6\Delta x_J}(X_{J+1} - x)^3 + \frac{d_{j+1}}{6\Delta x_J}(x - x_j)^3 + \frac{f_{j+1} - f_j}{\Delta x_J} + \frac{f_j x_{j+1} - f_{j+1} x_j}{\Delta x_J}$$
$$- \frac{\Delta x_J}{6}\left(d_{j+1}\left(1 - x_j\right) = d_j\left(1 - x_{j+1}\right)\right)$$

Since the first derivative is continuous and applies

$P_j(x_{j+1})=P_{j+1}(x_{j+1})$ will obviously fsatisfy $P_j(x_{j+1})=P_{j+1}(x_{j+1}$

That leads us to the system

$$\Delta x_{j-1} d_{j-1} + 2\left(\Delta x_{j-1} + \Delta x_j\right) d_j + \Delta x_j d_{j+1} = 6\left(\frac{f_{j-1}}{\Delta x_{j-1}} f_j \left(\frac{1}{\Delta x_{j-1}} + \frac{1}{\Delta x_j}\right) + \frac{f_{j+1}}{\Delta x_j}\right)$$

j=2,3,…,n-1

The d_1 and d_n are rendered. If $d_1 = d_n = 0$ then we call the polynomials $P_j(x)$ natural cubic splines functions

The system (4.11) can be written as Ad = F, where

$$A = \begin{bmatrix} 2(\Delta x_1 + \Delta x_2) & \Delta x_1 & 0 & 0 \ \dots & 0 \\ \Delta x_1 & 2(\Delta x_2 + \Delta x_3) & \Delta x_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \Delta x_{n-2} & 0 & 2(\Delta x_{n-2} \Delta x_{n-1}) & \Delta x_{n-1} \\ 0 & 0 & 0 & \Delta x_{n-1} & 2(\Delta x_{n-1} \Delta x_n) \end{bmatrix}$$

$$d = \begin{bmatrix} d_2 \\ d_3 \\ \vdots \\ \vdots \\ d_{n-1} \end{bmatrix} \qquad F = \begin{bmatrix} F_2 \\ F_3 \\ \vdots \\ \vdots \\ F_{n-1} \end{bmatrix}$$

$$F_j = 6\left(\frac{f_{j-1}}{\Delta x_{j-1}} - f_j\left(\frac{1}{\Delta x_{j-1}} + \frac{1}{\Delta x_j}\right) + \frac{f_{j+1}}{\Delta x_j}, \quad j = 2,3,\dots,n-1$$

The above system is triagonal and symmetric and has diagonal supremacy. To solve this system we use the method of Cholesky.

Program

This program calculates the coefficients d_j of functions splines. Program data is the number of points , the interpolation points x_j, the values of the function $f(x_j)$ and the intermediate point x we ask the approximate value f (x). The interval $[x_j,x_{j+1}]$ is automatically calculated by the program. Run the program with the same data the example of Langange program and we demand the value f (0.625). The answer is

The coefficients splines are

D (0) = 0 D (1) = 1.471421

D (2) = 1.467158

D (3) = 1.640696

D (4) = 2.939276

D (5) = 0

$f(0.625) = 1.867892$